Solar quadrupole moment and purely relativistic gravitation contributions to Mercury's perihelion Advance

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ABSTRACT: The perihelion advance of the orbit of Mercury has long been one of the observational cornerstones for testing General Relativity (G.R.).

The main goal of this paper is to discuss how, presently, observational and theoretical constraints may challenge Einstein's theory of gravitation characterized by $\beta = \gamma = 1$. To achieve this purpose, we will first recall the experimental constraints upon the Eddington-Robertson parameters γ, β and the observational bounds for the perihelion advance of Mercury, $\Delta\omega_{obs}$.

A second point will address the values given, up to now, to the solar quadrupole moment by several authors. Then, we will briefly comment why we use a recent theoretical determination of the solar quadrupole moment, $J_2 = (2.0 \pm 0.4)\ 10^{-7}$, which takes into account both surfacic and internal differential rotation, in order to compute the solar contribution to Mercury's perihelion advance.

Further on, combining bounds on γ and J_2 contributions, and taking into account the observational data range for $\Delta\omega_{obs}$, we will be able to give a range of values for β.

Alternatively, taking into account the observed value of $\Delta\omega_{obs}$, one can deduce a dynamical estimation of J_2 in the setting of G.R. This point is important as it provides a solar model independent estimation that can be confronted with other determinations of J_2 based upon solar theory and solar observations (oscillation data, oblateness...).

Finally, a glimpse at future satellite experiments will help us to understand how stronger constraints upon the parameter space (γ, β, J_2) as well as a separation of the two contributions (from the quadrupole moment, J_2, or purely relativistic, $2\alpha^2 + 2\alpha\gamma - \beta$) might be expected in the future.

KEYWORDS: celestial mechanics; planetary dynamics; orbits; Sun; Mercury, theory of gravitation, Eddington-Robertson parameters.

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1 INTRODUCTION

The solar quadrupole moment, J_2, is one of the fundamental figures in solar physics. It provides informations on the distortion of the effective solar potential, J_2 being the first perturbation coefficient to a pure spherically symmetric gravitational field:

$$\Phi(r,\theta) = -\frac{GM}{r}\left[1 - \sum_{n=1}^{\infty}\left(\frac{R_s}{r}\right)^2 J_n\ P_n(\cos\theta)\right]$$

where Φ is the solar component of the gravitational potential outside the Sun, in polar coordinates (r,θ,ϕ) with respect to the Sun's rotation axis; P_n are Legendre functions of degree n. The coefficients J_n are thus directly related to the distorted shape of the Sun; for instance, for $n = 2$, $J_2 \neq 0$ is an indicator of the oblateness[3].

Concerning the Sun, J_2, which is the most important term, should be used as a constraint in the computation of solar models, as the asphericity is a probe to test the solar interior.

Further, detection of long term changes in the solar figure (as there is some evidence for J_2 to vary with time) are intended; those have been postulated to act as a potential gravitational reservoir that can be a source of solar luminosity variations, which in turn, could have significant effects on the climate of the Earth ([So et al.1979], [Roz2001a]).

Today, the quadrupolar moment is also a non negligeable quantity in computing the relativistic motion of planets.

The first time that the solar quadrupole moment was associated with gravitational motion of Mercury is in 1885, when Newcomb attempted to account for the anomalous perihelion advance of Mercury with a modified gravitational field, manifested by an oblateness Δr [Ne1895-1898]. Indeed, in 1859, Le Verrier had observed a deviation of Mercury's orbit from Newtonian's predictions, that could not be due to the presence of known planets. But, the difference between the equatorial and polar diameters of the Sun of 500 arc ms, as advocated by Newcomb, was soon ruled out by solar observations. And Einstein's new theory of gravitation, General Relativity, could account for *almost all* the observed perihelion advance.

So, Mercury's perihelion advance readily became one of the cornerstones for

[3]Notice that $J_2 = -c_{02}$, where c_{02} is the second spherical harmonic coefficient.

testing General Relativity; even though, now, a contribution to the perihelion shift from the solar figure (though less important than first suggested by Newcomb) can not be discarded.

Mercury is the inner most of the four terrestrial planets in the Solar System, moving with a high velocity in the Sun's gravitational field. Only comets and asteroids approach the Sun closer at perihelion. This why the "Mercury lab" and minor planets too (see section 4.1.2) offer unique possibilities for testing G.R. and exploring the limits of alternative theories of gravitation with an interesting accuracy.

However, the perihelion shift of planets, and hence Mercury, can not be measured directly because the perihelion is a Keplerian element whereas the motions of the planets are not exactly Keplerian due to mutual gravitational interactions and figure effects. So, only an indirect determination can be done. One can proceed as follows. The motions of planets, from numerically integrated ephemeris, are computed over an interval of time. The time evolution of osculating elements is then plot and a polynomial fit of the parameters gives the rate of the perihelion advance. If one repeats this procedure in the classical Newtonian limit, one gets another set of rates. The difference between the two computations, and taking into account the constant general precession of the equinoxes, gives the combined effect due to relativistic gravitation and the Sun's quadrupole moment, $\Delta\omega_{obs}$.

Nevertheless, $\Delta\omega_{obs}$ depends on how the perturbation elements (for example the slow motion of the ecliptic) are taken into account in the computations [NaRa1985]; it also depends on the precision and on the data set selected from the radar data [Ra1987] which provide the core of the ephemerides computation.

Furthermore, in the article [Pit1993], the author shows that the topographic features of Mercury's surface influence the results on the perihelion advance inferred from Mercury radar observations.

These are the main reasons for which the range of Mercury's perihelion advance deduced from the radar data remains of a great amplitude (see table 1, section 7).

M. Standish, [St2000], has applied a method analoguous the above mentioned method; integrating equations over four centuries, 1800-2200, with and without the relativistic contribution (the second integration was done by simply replacing the speed of light with a very large value). He then computed the perihelion of Mercury from both of the runs, one point every 400 days, and differentiated the values of the perihelion at each time-point. After fitting

a linear function to the differences, the resulting slope from the figure is: 42.980 ± 0.002 arcsec/cy. Both integrations assumed $J_2 = (2.0 \pm 0.4)\ 10^{-7}$, hence the estimation of the perihelion advance given by M. Standish represents solely the purely relativistic contribution.
At such a level of precision, this solution would scarcely change with future ephemeris improvements (or with another set of ephemerides calculations, such as those given by the Bureau des Longitudes, since 1889 [Bureau1989]).

In the following second section, we will describe how, using the most accurate theoretical value for J_2 , the observed perihelion advance can lead to constraints upon the parameters (β, γ) which describe a generic, metric and conservative theory of gravitation.
In section 3, we will see how, in the setting of General Relativity, $\Delta\omega_{obs}$ can be used to provide a dynamical, solar model independent, estimation of J_2. This dynamical value can be confronted with that derived from direct measurements of the solar oblateness or indirect ones coming from helioseismology, which are solar model dependent.
Finally, we will give, in section 4, an overview of future satellite experiments that might be expected to put stronger constraints upon the parameter space (β, γ, J_2) in the future.
Throughout this article, we will refer to estimated values of $\Delta\omega_{obs}$ and J_2 from different authors and sources. Those are listed in tables at the end of this review, along with the figures, in section 7.

2 CONSTRAINTS UPON GRAVITATION THEORIES

2.1 The relativistic advance of the perihelion of Mercury

2.1.1 The purely relativistic effect.

Once correcting for the perturbation due to the general precession of the equinoxes (~ 5000 (arcsec/cy)) and for the perturbations due to other planets (computed numerically with a Newtonian N-body model: ~ 280(arcsec/cy) from Venus, ~ 150 (arcsec/cy) from Jupiter and ~ 100 (arcsec/cy) from the rest), the advance (in regards to the classical Keplerian prediction) of the perihelion of Mercury is a combination of a purely relativistic effect and a

contribution from the Sun's quadrupole moment.
It is given by the following general expression[4]:

$$\begin{array}{ll} \Delta\omega = \Delta\omega_{0\ GR}\ \delta \qquad \text{(rad/revolution)} & \\ \text{with}\quad \Delta\omega_{0\ GR} \equiv \frac{3\pi R}{\alpha\ a\ (1-e^2)} & (1) \\ \qquad \delta \equiv \left[\frac{1}{3}\left(2\alpha^2 + 2\alpha\gamma - \beta\right) - \frac{R_s^2}{R\ \alpha\ a(1-e^2)}\ J_2\ \left(3\sin^2 i - 1\right)\right] & \end{array}$$

and where the following parameters[5] are

R , the Schwarzschild radius of the Sun, $\frac{2GM_s}{c^2}$, given in [EPJ2000]; (2)

M_s , the Sun's mass, given in [All2000]; (3)

R_s , the Sun's radius, given in [EPJ2000]; (4)

J_2 , the quadrupole moment of the Sun for which we take the theoretical value of $(2.0 \pm 0.4)\ 10^{-7}$, in [GodRoz2000]; (5)

a , the semi-major axis of Mercury's orbit, in [All2000]; (6)

e , the exentricity of Mercury's orbit, in [All2000]; (7)

i , the inclination of Mercury's orbit, in [All2000]; (8)

Notice that formula (1) is only valid for fully conservative theories. If it is not the case, the complete expression is recovered with the following change

$$\delta = \left[\begin{array}{l} \frac{1}{3}\left(2\alpha^2 + 2\alpha\gamma - \beta\right) \\ -\frac{R_s^2}{R\ \alpha\ a(1-e^2)}\ J_2\ \left(3\sin^2 i - 1\right) \\ +\frac{1}{6}\left(2\alpha_1 - \alpha_2 + \alpha_3 + 2\zeta_2\right)\frac{M_s M_M}{(M_s+M_M)^2} \end{array}\right]$$

where "M_M" is the mass of Mercury; "α_1", "α_2", "α_3" parametrize preferred-frame effects; and "ζ_2", "α_3", a violation of the conservation of the total momentum.
But the extra term is nevertheless negligeable because it is proportional to $\frac{M_s M_M}{(M_s+M_M)^2} \sim \frac{M_M}{M_s} \sim 2\ 10^{-7}$ (see [Will1993]), and thus negligeable in regards

[4] In some references, the coefficient of the term containing the contribution of the orbit's inclination is improperly written.

[5] Notice that the value of the Schwarzschild radius of the Sun is more accurate than the separate values of the gravitational constant, G, and the Sun's mass, M_s.

to the first one (of the order of unity) or to the second one (of the order of 10^{-4}).

"α", "β" and "γ", refer to the Eddington-Robertson parameters of the Parametrized Post-Newtonian (P.P.N.) formalism, describing a fully conservative relativistic theory of gravitation.
"α" describes the weak equivalence principle; "β" is the amount of non-linearity in the superposition law of gravity; and "γ" characterizes the amount of space curvature produced by unit rest mass.
The P.P.N. parameters also cover the particular case of Einstein's theory of gravitation, General Relativity, characterized by $\alpha = \beta = \gamma = 1$.

2.1.2 Constraints upon the Eddington-Robertson parameters.

First of all, the parameter α is set to unity for any theory that respects the weak equivalence principle, well tested (the difference between the acceleration towards the Earth of two test-masses of different composition, relative to the sum of those accelerations, is inferior to $\sim 10^{-14}$. See [Will2001]).
Note that the Microscope Mission, selected by the French agency C.N.E.S. and scheduled for launch by 2004, has for scientific objective to test the equivalence principle up to an accuracy of 10^{-15}, using its well known manifestation, the universality of free fall [To et al.2000].

Secondly, it is light deflection experiments (measuring the combination $\frac{\alpha+\gamma}{2}$) that provide so far the best constrains on γ [Leb1995][6]:

$$\gamma = 0.9996 \pm 0.0017 \tag{9}$$

But there is, presently, no independent determination of the parameter β, which appears either in the combination $2\alpha^2 + 2\alpha\gamma - \beta$ characterizing the perihelion advance, or in the Nordtvedt effect $4\beta - \gamma - 3 \equiv \eta$ (see section 2.2.4).

2.2 Theoretical solar quadrupole moment contribution

[6] According to other authors, [Rob et al.1991], the value of this parameter deduced from V.L.B.I. measurements is $\gamma = 1.0002 \pm 0.00096$; whilts Eubanks et al., as quoted by [Will2001], give $\frac{1+\gamma}{2} = 0.99992 \pm 0.00014$ (not yet published).

2.2.1 The question of the accurate determination of J_2.

The evaluation of the solar quadrupole moment, J_2, still faces some controversy: on one side, the theoretical values strongly depend on the solar model used, whereas accurate measurements are very difficult to obtain from observations.

Concerning this last point, let us for example recall some problems:
(1) the real differences of brightness of the solar limb dependency on the latitude; influence of faculae, sunspots and magnetic fields; correlatively, real effects due to latitudinal variation of the solar limb darkening function;
(2) the questioned solar activity (solar cycle) dependency of the Sun's oblateness. These variations were first conjectured by Dicke et al. in 1985 [Di et al.1987]. Observations at the Pic du Midi Observatory (France) from 1993 till 2000 seem to confirm a faint variability reported in previous observations made in 1983-1984. Nevertheless, the amplitude of the observed variations does not exceed 0.02" $-$ 0.04" over 20 years. (From [Ku1998]. See also in [Roz1996], fig. 1 and 2; in [RozRö1996], and in [RozRö1997] where the authors derive from all the available data a maximum value of J_2 of $1\ 10^{-5}$ and an average value of $(3.64 \pm 2.84)\ \ 10^{-6}$);
(3) and the difficulty to calibrate ground data in regards to atmospheric disturbances (local atmospheric refractive indexes and distortions due to atmospheric waves).

Space experiments have been suggested in order to solve those problems; however, first results obtained from the SoHO mission, [Ku et al.1998], have established a good concordance with ground-based observations of the oblateness (and thus J_2). Further comments are found in section 4.4.

To illustrate those difficulties, we give a compilation of the main determinations of J_2, based on observations and solar theory, in addition to the main critics to the method used (see table 2 and figure 2, section 7). A more detailed historical review can be found in [Roz1996] or in [RozRö1997]. Remark that early estimations of J_2, before 1967, using an heliometer of photographic plates, often erroneously predicted a prolate Sun (see the second table in [WitDeb1987]).

In this context, we see that a dynamical determination of J_2, using the perihelion shift of Mercury, is interesting as it might be confronted to those derived from solar model dependent values of the oblateness (see section 3).

2.2.2 The adopted theoretical value of J_2.

The theoretical value of J_2, used in this article, has been deduced from a recent work, where the authors have applied a "differential theory" to a solar stratified model, taking into account the latitudinal differential rotation. The result is a determination of J_2 as $(1.60 \pm 0.04)\ 10^{-7}$ at the surface of the Sun (see [GodRoz1999a] and [GodRoz2000]).
The value obtained is in agreement with those calculated by Paternò [Pa et al.1996], $J_2 = (2.22 \pm 0.1)\ 10^{-7}$, and Pijpers [Pij1998], $J_2 = (2.18 \pm 0.06)\ 10^{-7}$, using in their computations the inversion techniques applied to helioseismology.
The slight difference between these values and those of Godier/Rozelot comes mainly from the incertitudes on the solar rotation data due to the analytical rotation law adopted by [GodRoz1999b] (which gives a low velocity rate at the equator a bit lower than what is currently observed). But this difference does not question the order of magnitude[7], 10^{-4}, of the solar contribution in Mercury's perihelion advance. This is why we have admitted the theoretical range $(2.0 \pm 0.4)\ 10^{-7}$ for J_2.
This value can be confronted to the one given by other authors in table 2 or figure 2 (section 7).

2.2.3 G.R.'s prediction with and without the quadrupole moment contribution.

Using the values of the parameters given in the appropriate references (see (2), (3), (4), (5), (6) and (7)) in (1), plus the value of the period of Mercury's orbit given in [All2000], one finds

$$\Delta\omega_{0\ GR} = \frac{6\pi G M_s}{a\,(1-e^2)\,c^2} = 42.98\mathit{1} \qquad \text{(arcsec/cy)} \tag{10}$$

for which the accuracy is on the last digit.
This is the prediction of the perihelion shift of Mercury in the setting of G.R. theory, but omitting the contribution of J_2.
This raw value is excluded by the last observational data given by [An1992],

[7] Excluding the unacceptable estimations, that till recently, reported J_2 to be as large as 10^{-5} (see table 2), an order of magnitude larger than the theoretical upper limit allowed by lunar librations [RozBo1998].

[St2000], [Pit2001a], but not by [Kr et al.1993] and [Pit1993] (see table 1, section 7)

But, once the quadrupolar correction is added, using (8), this leads to

$$\Delta\omega_{GR} \in [43.00\mathit{0}\ ;\ 43.01\mathit{0}] \qquad \text{(arcsec/cy) for } J_2 = (2.0 \pm 0.4)\ 10^{-7} \qquad (11)$$

which is now consistent with the observations given by [An1992], [St2000], [Pit2001a], while still in agreement with [Kr et al.1993] and [Pit1993] (see table 1, section 7).
This last result also shows that the theoretical prediction for J_2, argued by the authors, is coherent with observations in the setting of G.R.

An important remark on values adopted for G.R.'s prediction of Mercury's perihelion advance, $\Delta\omega_{0\ GR}$, in the past is given in [NoWill1986]. The authors also interestingly underline the following fact: *"Although of theoretical interest, the difference between these quoted predictions for Mercury's perihelion advance has no observational consequence (for present methods of evaluation of Mercury's perihelion shift)"*. Indeed, the predicted general relativistic contribution to Mercury's perihelion advance is not an input in current procedures testing gravitational theories with the dynamics of Mercury. In modern ephemeris used to compute the motion of Mercury, the equations of motion already include relativistic post-Newtonian terms which are non-periodic. Those contribute to the secular variation of the orbital elements, among which, the perihelion of Mercury. The post-Newtonian terms in the ephemeris are modulated by a set of parameters (P.P.N. parameters describing the gravitational theory, masses or initial conditions of the planets, ...) that become part of a multiparameter least-squared fit to the observational data (radar, optical data ...) in order to obtain an improved determination of the parameters in the least-squares sense.
However, it is impossible, presently, to fit simultaneously for both the P.P.N. parameters and J_2, the two contributions, relativistic and Newtonian respectively, to the perihelion shift being too correlated in the case of Mercury alone (see section 4.1.2). Thus, one can either directly test (fit) the P.P.N. parameters assuming a given input value for J_2 in the ephemeris; or assume G.R. as the gravitational theory and test J_2. In the last case, expression (1) together with (10) are useful to provide a value of J_2, once $\alpha = \beta = \gamma = 1$ is assumed.
Nevertheless, the real general relativistic prediction for the perihelion shift of Mercury ($\Delta\omega_{0\ GR}$) is given unequivocally by (10), according to present values of astrophysical constants.

2.2.4 Alternative theories to G.R. gravity.

General Relativity is often considered today as "THE" relativist theory of gravitation. This pure tensor theory corresponds to a Newtonian potential that evolves as $1/r$, r being the radial coordinate. The theory so far agrees with all the observations made in our solar system. *Nevertheless, the theory of General Relativity can not be the final theory describing gravitation.*

First of all, *from a theoretical point of view*, General Relativity can not be quantified, and this makes it impossible to unify it with other fundamental interactions.
Moreover, the minimal choice of the Hilbert Einstein action, to which G.R. corresponds, is not based upon any fundamental principle. Or to express it in another way, it is evident that covariance and Newtonian fields approximation alone do not determine uniquely the action. Equivalently, nothing guaranties that the Newtonian potential is truly universal.
Any other theory of gravitation would be valid too, as long as it would lead to the same predictions as G.R. that have been tested in the solar system, with maybe some departures from the Einsteinian theory on larger distance scales.
Also, let us warn that, from the formal point of view, the theory of General Relativity is not invariant under conformal transformations. While, if we wish to achieve the junction between particle physics, in which conformal invariance plays a crucial role, and gravitation, we should consider a theory of gravitation that incorporates this property.

From the experimental point of view, let us notice that General Relativity alone still can not reproduce the flat velocity distributions in the vicinity of galaxies. The Newtonian potential would indeed predict a decreasing distribution.
We are thus confronted to the following dilemma: either we suppose the existence of dark matter, either we modify the potential for galactic distances. This second solution would immediately invalidate General Relativity with a null cosmological constant.
Let us also remark that the solution to the "dark matter dilemma" could also be a combination of the two solutions cited here above.

In conclusion, according to the above arguments, it is fundamental to conceive that alternative theories to General Relativity, that is to say $\beta \neq 1$ or/and $\gamma \neq 1$, are truly not excluded by the observations... as the case of G.R., $\beta = \gamma = 1$, is only a particular spot in the allowed parameter space

(β, γ, J_2).
This is illustrated by plotting ellipses representing the 1σ, 2σ and 3σ confidence levels in the (β, γ) plane, owing to Mercury's perihelion advance test, for a fixed value of J_2 (See Figure 1 a, b, c). Further, adding constraints on γ and β coming from tests of the Nordtvedt effect and L.L.R. data (see (9) and (12)) allows to select a portion of the ellipses in the (β, γ) plane. However, G.R always belongs at least to the 3σ region in the allowed parameter space (β, γ), according to the theoretical bounds on J_2 adopted by the authors (see section 2.2.2).
Nevertheless, we can conclude that $\beta = 1$ is not the only allowed case.

Looking more in details at the contribution of β to the perihelion shift, we see that the deviation, $1 - \beta$, from G.R.'s value, is, owing to the error bars, of the same order of magnitude as the contribution of J_2.
Thus two cases may be envisaged:
Either $\beta < 1$, which means that $\Delta\omega$ tends to be larger than $\Delta\omega_{GR}$, and the effect of $1 - \beta$ adds to the contribution of J_2.
Either $\beta > 1$, which means that $\Delta\omega$ tends to be smaller than $\Delta\omega_{GR}$, and the effects of $1 - \beta$ and J_2 substracts.

A possible consequence of β being different from unity is the Nordtvedt effect. Indeed, as soon as the combination of the Eddington-Robertson parameters given by $\eta \equiv 4\beta - \gamma - 3$ is non null, the gravitational and inertial masses of a celestial body are no longer the same (see [Will1993] and [Will2001] and references there in).

New analysis of the Lunar Laser Ranging data (L.L.R.) by [Willi et al.2001] provides $\eta = +0.0002 \pm 0.0009$, from which one may deduce the acceptable range for β, using the value of γ given by (9):

$$\beta \in [0.9993; 1.0006] \ . \tag{12}$$

This in turn allows us to infer a theoretical shift, $\Delta\omega$:

$$\Delta\omega \in [42.93\mathit{2}\ ;\ 43.05\mathit{7}] \qquad \text{(arcsec/cy) for } J_2 = (2.0 \pm 0.4)\ 10^{-7} \ .$$

We can see that it of course contains the particular case of G.R. ($\Delta\omega_{GR}$), and that it is consistent with recent observations ($\Delta\omega_{obs}$) (see table 1, section 7).

Alternatively, owing to the remark made in section 2.2.3, the fit of the most recent ephemeris EPM2000 to accurate ranging observations concerning the motion of planets (and in particular, the perihelion shift of Mercury),

provide an astonishingly precise estimation of the Eddington-Robertson parameters β and γ for a given theoretical value of J_2.
Indeed, according to reference [Pit2001b]:

$$\beta = 1.0004 \pm 0.0002 \text{ and } \gamma = 1.0001 \pm 0.0001. \tag{13}$$

for a theoretical value of 2.0 10^{-7} for J_2, in agreement with (5). However, the uncertainties upon the obtained parameters β and γ are formal deviations, and realistic error bounds may be an order of magnitude larger. Moreover, this estimation is rather tolerant regarding the assumed value of J_2. Indeed, $\beta = 1.000 \pm 0.001$ and $\gamma = 1.0005 \pm 0.0002$ have been obtained using the test ephemeris which only differ from EPM2000 by the solar oblateness $J_2 = 0.0$ [Pit2001a]!

3 Inferring a dynamical value of the solar quadrupole moment in the setting of G.R.

Conversely, one may think to infer the absolute value of the quadrupole moment, J_2, which is necessary (owing the allowed parameter space for β and γ) to be in agreement with observations. But, as mentioned in a remark in section 2.2.3, the purely relativistic contribution $(2\alpha^2 + 2\alpha\gamma - \beta)$ and the quadrupolar moment of the Sun (J_2) are too correlated in the perihelion advance of Mercury to lead simultaneously to interesting constraints on (β, γ) and J_2 separately.
This is why, so far, a dynamical estimation of J_2 is made in the setting of G.R.

In the particular case of G. R., the theory parametrized by $\alpha = \beta = \gamma = 1$, we find the results listed in table 3 (section 7) inferred from $\Delta\omega_{obs}$ (table 1, section 7) using equation (1).

Nevertheless, J_2 may not exceed the critical theoretical value of 3.0 10^{-6} according to the argument given in [RozBo1998][8], based upon the accurate knowledge of the Moon's physical librations, for which the L.L.R. data reaches accuracies at the milli-arcsecond level.
Moreover, J_2 has to be positive to be in agrement with an oblate Sun. So,

[8] This estimation does not take into account a possible temporal dependence of J_2. If such a variability exists, the amplitude is, nevertheless, obviously upper bounded by the critical value of 3.0 10^{-6}.

only Standish and Pitjeva's last results, [Pit1993], [St2000] and [Pit2001b], give interesting dynamical constraints upon J_2 (for the other authors, the error bars are too large or J_2 is negative, in contradiction with an oblate Sun). Namely: $J_2 \leq 2.89\ 10^{-7}$ for the EPM1988 ephemeris model, $J_2 \leq 3.38\ 10^{-7}$ for the DE200 model, $J_2 = (1.90 \pm 0.16)\ 10^{-7}$ for the DE405 model and $J_2 = (2.453 \pm 0.701)\ 10^{-7}$ for the more recent EPM2000 model (see table 3, section 7). Those values are compatible with the solar model dependent theoretical value of J_2, (5), argued by the authors in section 2.2.2.

4 Increasing precision in the future

4.1 From Hipparcos to GAIA satellite: towards an astonishing precision upon (γ, β, J_2)

4.1.1 Light deflection: γ

Milli-arcsec astrometry is available since 1996 from Hipparcos satellite data. The reduction of this data required the inclusion of stellar aberration up to terms in v/c^2, as well as the correction (in $\frac{\alpha+\gamma}{2}$) due to the relativistic light deflection in the gravitational field of the Earth and the Sun. Calculations for the Hipparcos data were implicitly made in the setting of G.R. ($\alpha = \beta = \gamma = 1$), thus allowing for this theory to be checked with a precision of $3\ 10^{-3}$ on γ. This is of course less accurate than the results based on V.L.B.I. measurements [Rob et al.1991], but Hipparcos opened the door to future micro-arcsec astrometry, which can improve the precision upon γ by several orders of magnitude.
Indeed, in the observational context of light deflection, the satellite GAIA [GAIA2000], one cornerstone of ESA's Space Science Programme, to be launched in 2009 (or at least no later than 2012) for a five years mission, will increase the domain of observations by two orders of magnitude in length (now, light deflection is tested on distances ranging from 10^9 to 10^{21} m) and six orders of magnitude in mass (now 1 to 10^{13} M_s). Moreover, GAIA, improving Hipparcos' performance, will reduce the avoidance angle towards the Sun, thus allowing to measure stronger light deflection effects with a reduced parallax correlation.
This all results into an estimated accuracy of $5\ 10^{-7}$ on γ.

Notice that the quadrupolar moment of the Sun, J_2, has a contribution

to the light deflection that is negligeable in the case of GAIA, owing to its non null avoidance angle[9].

4.1.2 Perihelion precession for minor planets: $2\alpha^2 + 2\alpha\gamma - \beta$ and J_2

GAIA is also expected to observe and discover several hundred thousand minor planets, mostly from the Main Belt.
All of them will acknowledge a perihelion shift (see equation (1)), just like Mercury, but with a magnitude in respect to the eccentricity, e, inclination, i, and semi-major axis, a, of their own orbit.
Thus, the relativistic correction *per revolution* to the orbital motion will only be significant for the Apollo, Aten and Armor groups, which means of the same order of magnitude[10] as for Mercury. (In contrast, it will be about seven times smaller for minor planets of the Main Belt). But unlike the Apollo and Aten groups, the Armor group are not Earth-Orbit crossers.
On the other side, *the absolute precession rate* will be approximately four times bigger for Mercury than for members of the Apollo or Aten groups owing to their respective revolution periods (and more than 100 times bigger for the population of the Main Belt).
Remark that the perihelion shift of the minor planet Icarus had already been used in the past (as early as in 1968 [LiNu1969]) in order to infer a dynamical value for J_2. But the non uniform distribution of earlier observations[11] over the orbit of Icarus and Earth seriously affected the suitability of (just) Icarus data in verifying G.R. or estimating J_2 independently (see [Sh1965], [Sh et al.1968] and [Sh et al.1971]). So the estimations of J_2 were obtained assuming G.R. (see table 4, section 7).

The advantages of measuring the perihelion shift of minor planets with GAIA, in addition to Mercury's, are multiple.
First, there will be, of course, an increased precision on individual determinations of $\Delta\omega$, due to GAIA's technology but also to the fact that minor planets are not as extended as Mercury, and so their position can be mea-

[9] In the case of planets like Saturn or Jupiter being the deflector, the contribution of $J_{2\ planet}$ to the light deflection effect is non negligeable, due to the important magnitude of $J_{2\ planet}$ and to the fact that grazing incidence is allowed.

[10] For some exemples see [GAIA2000], page 116 table 1.18.

[11] Based on photographic observations from 1949-1968, for [Sh et al.1971], plus 7 Doppler-shift observations for reference [LiNu1969]; and additional observations during the encounter with Earth in 1987 for reference [La1992])

sured more precisely.
Secondly, a statistic on several tens of planets, which is a statistic on $\Delta\omega(a,\ e)$ (or $\delta(a,\ e)$), will allow to increase the accuracy on the determination of J_2 and the combination "$2\alpha^2 + 2\alpha\gamma - \beta$" separately. Remember that those two contributions have different dependencies in "$a\ \ (1 - e^2)$" [Gou1982].
Thirdly, by studying the precession of the orbital plane of a minor planet about the Sun's polar axis, due to the quadrupolar moment of the Sun (J_2) but unaffected by relativistic gravitation ($2\alpha^2 + 2\alpha\gamma - \beta$), one should be able to dynamically measure J_2 independently. This effect being more easily discernible for moderately large values of the inclination, i, minor planets like Icarus with a large value of i ($i \simeq 16°$) would be truly adequate ([Di1965], [Sh1965], [Sh et al.1968]).

A dedicated simulation still has to be performed to assess the real capabilities of GAIA in that field. But, so far, an estimation of a precision of 10^{-4} on the combination "$2\alpha^2 + 2\alpha\gamma - \beta$" from individual determinations of $\Delta\omega$, seems reasonable; moreover, 10^{-5} should be attainable thanks to statistics on several tens of planets.

From the point of view of J_2, GAIA should be more precise than 10^{-7}, but the accuracy is difficult to assess without an extensive simulation on the available sampling of "$a\ \ (1 - e^2)$". Nevertheless, through measurement of perihelion advances, GAIA should provide a more accurate dynamical and solar model independent determination of J_2, to be confronted with solar model dependent predictions from, for example, helioseismology data.

4.1.3 Resulting constraint on β

Using the independent constraint upon γ obtained by GAIA from light deflection, and the constraint upon "$2\alpha^2 + 2\alpha\gamma - \beta$" from perihelion shifts measured by GAIA, one should be able to constraint β with a precision of $3\ 10^{-4} - 3\ 10^{-5}$. This is about two orders of magnitude better than the present best determinations due to L.L.R.(see equation (12)) or direct fits of the data on the detection of η [Willi et al.2001].

4.2 Alternative future direct measurements of γ

GAIA will probably not deliver any results on γ before the end of its mission, but, in the mean time, other space or ground based measurements like V.L.B.I.'s, will certainly improve the present determination of γ. See table 5

(section 7) for proposed space missions purely dedicated to the measurement of γ.

As an illustration of further prospects, we can cite the Astrodynamical Space Test of Relativity using Optical Devices (ASTROD) [Bec et al.2000], a proposal that has been submitted to ESA in response to a "Call for missions proposals for two Flexi-Missions", but which is not yet accepted. Such mission, using time-delay measurements between two spacecrafts orbiting the Sun and the Earth, would certainly lead to precisions of the order of $10^{-6} - 10^{-7}$ on γ. But if the stability of the clocks/lasers can be reduced to 10^{-18}, then, using the range data of ASTROD as an input for a better determination of the solar and planetary parameters, one might dream to get a precision of $10^{-8} - 10^{-9}$ on γ! Moreover, from the precise determination of orbits, information could be given on the solar quadrupole moment, higher moments and β. Again, providing ultra-stable clocks, precisions of the order of 10^{-6} on β and $4.5\ 10^{-8}$ on J_2 could be reached!

4.3 A Mercury Orbiter mission: measuring β independently from γ as well as separating $\left(2\alpha^2 + 2\alpha\gamma - \beta\right)$'s contribution from J_2's

Scheduled to be launched in 2007 (or 2009) for a 2-5 years mission, BepiColombo, has been accepted as E.S.A.'s Cornerstone Mission #5 in 1996 [Bepi2000].
It contains three spacecraft elements, among which a Mercury Planetary Orbiter that will considerably help to reduce the error bars on the Eddington-Robertson parameters β, γ and the Sun's quadrupole moment J_2.
Indeed, an estimation of the accuracies attainable has been done thanks to a full simulation of radio science experiments with calibration of solar plasma noise, non gravitational accelerations and systematic effects.
The measurement of β, γ, J_2 and η is the output of a complex orbit determination process in which radio-metric and calibration acquired during the mission are used to provide a complete orbital solution which includes the osculating orbital elements of the spacecraft and the planet Mercury, as well as the harmonic coefficients of the planet's gravity field (at least to the degree and order 25). Indeed, precision range and range-rate measurements of BepiColombo will constrain the position of the planet's center of mass with a precision of about 1 m! Thus allowing a truly precise knowledge of the

orbital elements and the secular perihelion shift of Mercury in particular.
An additional advantage of a Mercury Orbiter, regarding the measure of the perihelion shift, is that it would considerably reduce the time scale by comparison to recent data observations that use time averages on a decade time scale. This would permit an eventual observation of the variation of the perihelion rotation due to a possible variation of the solar quadrupole moment J_2.

From the view point of constraining the Eddington-Robertson parameter γ, time delay measurements of radio signals travelling from the spacecraft to the Earth and back, combined with Doppler shift measurements of photons should help get a yet more precise determination of γ. BepiColombo should then be able to improve over Cassini's expected results (see table 4, section 7), thanks to frequent solar conjunctions during which the Doppler shift effect is maximum. A preliminary analysis of the mission estimates that γ could be accurately determined at 2.5 10^{-6} [Bepi2000].

The precise determination of Mercury's motion would also help measure the Nordtvedt effect, η, with an expected accuracy of 2 10^{-5}. This combined with the values found for the perihelion advance $(2\alpha^2+2\alpha\gamma-\beta)$ and γ would help lift the degeneracy between β and J_2.

Probing the gravitational field of Mercury at various distances from the planet would also help separate the effects of J_2 from those of relativistic gravitation $(2\alpha^2+2\alpha\gamma-\beta)$, owing to their different dependency in the radial distance to Mercury [Will1993].
Following this idea, advantage can be taken of the large eccentricity of Mercury's orbit to search for periodic orbital perturbations induced by J_2 and relativistic gravity [Gou1982], [Will1993].
This is how J_2 would be determined independently by the BepiColombo Mission: from the precise determination of the secular nodal precession of the planet's orbital plane about the Sun's polar axis, due to the quadrupolar moment of the Sun, but unaffected by purely relativistic gravitation.
All this should lead to a determination of J_2 with a precision of 2 10^{-9}, (see [Tu et al.1996] and [Bepi2000]).

Notice that the influence of Mercury's topography in determining the perihelion precession has been stressed by some authors [Pit1993], [Pit2001b]. It has to be taken into account when processing radar observations, as it might help to reduce the systematic errors in ranging. So far, the scarcity of radar observational data for Mercury restricts the accuracy of estimates for the topographical contribution. But future Mercury orbiters, like BepiColombo

or N.A.S.A.'s discovery mission named Messenger[12] (to be launched in 2004), could remedy to that problem by providing useful complementary data upon the topography of the planet.

Finally, we shall cite an interesting proposal from article [Tu et al.1996] (page 24, equation 40), that suggests a measure of β independent of γ, testing the strong equivalence principle, using a Mercury orbiter on a particular resonant orbit.

4.4 Satellites dedicated to J_2

Future solar probes are expected to determine a more precise value for J_2.

Indeed, the quadrupole moment of the Sun can be measured dynamically by sending and accurately tracking a probe, equipped with a drag-free guidance system, to within a few solar radii of the solar center. J_2 is then inferred from the precise determination of the trajectory.

Alternatively, J_2 can be inferred from in orbit measurements of solar properties. But in this case, the reduction of such a measurement will require a better understanding on how solar density models and rotational laws influence the multipole expansion of the external gravitational field [UlHa1981].

For example, the micro-satellite Picard is a C.N.E.S.[13] mission, due for flight by the end of 2005. The expected mission lifetime is 3 to 4 years with a possible extension to 6 years.

The aim of Picard [Da et al.2001], is to perform in orbit simultaneous accurate and absolute measurements of the solar diameter, differential rotation and irradiance, in addition to low frequency helioseismology, as a permanent viewing of the Sun from a G.T.O. orbit should allow the detection of g-modes. Picard should be able to measure J_2 with of precision of 10^{-8}.

Notice also that the diameter measurements will be obtained at any latitude (sunspots and faculae at limb removed) which should allow the detection of a latitudinal variation of the diameter and thus, the quadrupole moment, as predicted by the theoretical model used by the authors [GodRoz1999b].

Moreover, Picard will observe the Sun in different wavelength bands, among

[12]But, unlike BeppiColombo, it will not test G.R. nor measure the perihelion precession... (see http:\\discovery.nasa.gov\messenger.html and section 9.4 in reference [Bepi2000]).

[13]Centre National d'Etudes Spatiales (France)

which the band used for measurements of the Sun's diameter from the ground, 535.7 nm. This will permit one to compare space-measurements with ground-based ones in order to correct for atmospheric perturbations, and so, to eventually re-calibrate the existing data over the former solar cycle dependencies. Thus, the Picard mission is clearly designed to solve some of the problems mentioned in section 2.2.1 for the determination of J_2.

5 CONCLUSIONS

We have seen in this article that, **so far, the theory of General Relativity is not excluded by observations**. However G.R. only represents one possible point in the still large allowed parameter space (β, γ, J_2) and alternative theories are also permitted.
More precisely:

Future space experiments cited in the last section of this article will considerably reduce the parameter space in the near future, and so impose an even more rigorous test to G.R..

As a determination of the solar quadrupole moment is concerned, **we have stressed the importance of confronting a dynamical determination of $\mathbf{J_2}$, independent of the solar model (obtained from perihelion advances or motion of spacecrafts), to other solar model dependent values**.
Presently, this dynamical determination of J_2 is still dependent upon the gravitational theory $(2\alpha^2 + 2\alpha\gamma - \beta)$, but the **future GAIA or Bepi-Colombo missions should be able to separate those two effects, and so obtain a determination of $\mathbf{J_2}$ that is independent of the gravitational theory**.
So far too, the authors can say that their estimated theoretical value of $J2 = (2.0 \pm 0.4)\ 10^{-7}$, which is solar model dependent, together with the estimated error bars, is completely coherent with the estimated dynamical value resulting from Mercury perihelion advance in the setting of General Relativity.

We conclude by reminding that, if future observations confirm the time dependency of $J2$ (for example, a periodicity with the sunspots cycle), this effect will have to be taken into account when using perihelion advance as a test for G.R. (or alternatively, to estimate $J2$). This so far has not been the case, as dynamically estimated values of $J2$ come from a mean over several

decades (see for example [La1992]).

6 ACKNOWLEDGMENTS

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7 TABLES AND FIGURES

- Table 1 -

Inferred correction to G.R.'s prediction for Mercury's perihelion advance

References	Perihelion Advance $\Delta\omega_{obs}$ (arcsec/cy)	$\Delta\omega_{obs} - \Delta\omega_{0\ GR}$ (arcsec/cy)	$\delta = \Delta\omega_{obs}/\Delta\omega_{0\ GR}$
[Ne1895-1898]	~43.37	~0.39	~1.01
[Cl1943]	42.84±1.01	-0.14±1.01	0.997±0.023
[Cl1947]	42.57±0.96	-0.41±0.96	0.990±0.022
[Dun1958]	43.10±0.44	+0.12±0.44	1.003±0.010
[Way1966]	43.95±0.41	+0.97±0.41	1.023±0.010
[Sh et al.1972]	43.15±0.30	+0.17±0.30	1.004±0.007
[MoWar1975]	41.90±0.50	-1.08±0.50	0.975±0.016
[Sh et al.1976]	43.11±0.21	+0.13±0.21	1.003±0.005
[An et al.1978]	43.3±0.2	+0.32±0.2	1.007±0.005
[Bre1982a] [NaRa1985]	45.40±0.05	+2.42±0.16	1.056±0.001
[Bre1982b] [Ra1987]	45.25±0.05	+2.27±0.05	1.053±0.002
[Kr et al.1986]			
EPM1988	42.83 ± 0.12	-0.15 ± 0.12	0.997 ± 0.003
DE200	42.77 ± 0.12	-0.21 ± 0.12	0.995 ± 0.003
[Ra1987]	45.47±0.09	+2.49±0.09	1.058±0.002
[An et al.1987]	42.92±0.20	-0.06±0.20	0.999±0.005
[An1991]	42.94±0.20	-0.04±0.20	0.999±0.005
[An1992]	43.13±0.14	+0.15±0.14	1.003±0.003
[Kr et al.1993] :			
EPM1988	42.985 ± 0.061	+0.004 ± 0.061	1.0001 ± 0.0014
DE200	42.978 ± 0.061	-0.003 ± 0.061	0.9999 ± 0.0014
[Pit1993] :			
EPM1988	42.964 ± 0.052	-0.017 ± 0.052	0.9996 ± 0.0012
DE200	42.970 ± 0.052	-0.011 ± 0.052	0.9997 ± 0.0012
[St2000]			
DE405	43.004±0.002	+0.023±0.002	1.00054±0.00005
[Pit2001a] :			
EPM2000	43.0115±0.0085	+0.0305±0.0085	1.00071±0.00020

Table 1. In the past, planetary motions, necessary to infer the value of $\Delta\omega_{obs}$, were modeled with classical analytical theories. Presently, more precise space experiments require accurate numerical ephemeris. Those are made possible thanks to new astrometric methods (radar ranging, Lunar Laser Ranging, V.L.B.I.. measurements) that led to ranging data uncertainties of only a few meters.
An interesting review of EPM and DE numerical ephemeris can be found in reference [Pit2001b]. It discusses the history of planetary motion modeling and describes which data (optical, radar, L.L.R. or from space-craft) has been used for the different ephemeris.

Following this historical path, references [Ne1895-1898], [Cl1943] (calculated for Julian year J1900.00), [Cl1947] (J1850.00) and [Bre1982a] (J1900.00) used an analysis of the observed data on Mercury which was biased by the assumed theory of gravitation.
In [Cl1943], [Dun1958], [Way1966] and [MoWar1975], the same old analytical perturbation theory as Newcomb's, [Ne1895-1898], was used. Or, in [Cl1947], the results are based on Doolittle's calculations of the Newtonian motion, with certain corrections [Do1925]. Those methods were not as adequate as present numerical computations. While the semi-analytical theory developed in [LesBre1982] did not use sufficiently accurate equations of motions of planets.
See arguments given in [NaRa1985] and [Ra1987].

To each reference in table 1 corresponds a value of the advance of Mercury's perihelion deduced from observational data: $\Delta\omega_{obs}$.
For each of them, one may compute the possible corrective factor, δ, to the prediction due to Einstein's Gravity (G.R.), $\Delta\omega_{0\ GR} = 42.98\mathit{1}$ arcsec/cy, which does not include the quadrupolar (J_2) correction.
Notice that light deflection measurements constraint γ to 0.9996 ± 0.0017 [Leb1995]; while theoretical predictions for the solar quadrupole moment, taking into account surfacic and internal differential rotation, give $J_2 = (2.0 \pm 0.4)\ 10^{-7}$, which means that the solar correction to the perihelion advance, $-\frac{R_s^2}{R\ \alpha\ a(1-e^2)}\ J_2\ \left(3\sin^2 i - 1\right) = 2.821\mathit{8}\ 10^{-4}\frac{J_2}{10^{-7}}$ is $2.821\mathit{8}\ 10^{-4}\ (2.0 \pm 0.4)$.
New L.L.R. data [Willi et al.2001], on their side, provide
$\beta \in [0.9993; 1.0006]$.

Here follow some comments on how the values, $\Delta\omega_{obs}$, given in table

1, are inferred from the given references:

In [Cl1943], the discordance O-C between observations and the modeling theory used by Clemence is $-0".07 \pm 0.41$. In this theory, Clemence took 42".91 for the general relativistic contribution to the precession. $\Delta\omega_{obs}$ is thus the sum of those two numbers, where Clemence's estimation of the probable error in the uncertainty in the masses, ± 0.6, is taken into account.

The values given in this table for [Cl1947] result from Clemence's value $42".56 \pm 0.94$ (which is the difference between the total perihelion precession observed, $5599".74 \pm 0.41$, and the Newtonian contribution of the other Planets plus the effect of the solar oblateness, $5557".18 \pm 0.85$) from which Clemence's erroneous estimation of J_2's contribution, $0".010 \pm 0.02$, has been removed.

As reference [Way1966] is concerned, $\Delta\omega_{obs}$ is obtained by substracting the total Newtonian contribution of planets, 531".26 (that had been recalculated by the author of [Way1966] using Marsden's masses for planets) and the precession of the equinoxes as calculated by Newcomb [Ne1895-1898], 5024".53, from the total precession observed, 5599".74, as cited from [Cl1943].

The values given for [Sh et al.1972] are obtained from their given estimation of δ (written in their article as λ_p with $J_2 = 0$) that includes a correction for a typical representation of the topography of the Planet Mercury, using equations (1) and (10).

In references [Bre1982a] and [Bre1982b], the perihelion advance had been recalculated at J1900.00, using contemporary values for planetary masses (see [NaRa1985] and [Ra1987] respectively). For them, the constant rate of the perihelion advance due to the equinoxes precession is taken to be respectively $5029^{"}.0966 + 2.2223T$ (cited in [Bre1982a] from [Li et al.1977]) and $5029^{"}.0966 + 2.2274T$ (cited in [Bre1982b] from [BreCh1981]), where T is in Julian centuries with reference to the initial epoch J2000; and the N-body Newtonian contribution from planets, $528^{"}.95$, from [NaRa1985].

Notice that in the following articles, [Kr et al.1986], [Kr et al.1993], [Pit1993], the authors improperly writes 42.95 (arcsec/cy) for $\Delta\omega_{0\ GR}$. But the values for $\Delta \stackrel{\bullet}{\pi}$ given in those articles are not affected (see

remark in section 2.2.3). Indeed, the correction to the perihelion motion, written as $\Delta \overset{\bullet}{\pi}$ in those articles, has been obtained by fitting the EPM88 and DE200 numerical ephemerides to all radar observational data (from 1964 to 1989 for Pitjeva's) with the needed parameters (the elements of planetary orbits, the radii of planets, the value of AU, etc.). The correction to the perihelion motion is thus the "observed" deviation from the value of Mercury perihelion advanced obtained from EPM88 ephemerides with zero J_2 assumed on the time interval mentioned. In this present table, we computed $\Delta\omega_{obs}$ as $\Delta\omega_{EPM1988}+\Delta \overset{\bullet}{\pi}$, where is $\Delta\omega_{EPM1988}$ the mean value of the perihelion advance obtained from EPM1988 ephemerides, namely, 42".9806 $\simeq$ 42".981 (arcsec/cy).
As far as the estimation of the quadrupole moment, J_2, is concerned, it has been incorrectly done, in those articles, with the value 42".95 (arcsec/cy). But it can be recomputed afterwards; assuming G.R. (fixed value for $\alpha = \beta = \gamma = 1$) and using the differentiation of formula (1) with the proper value for $\Delta\omega_{0\ GR}$ given in (10):
$\Delta J_2 = \frac{\Delta\overset{\bullet}{\pi}}{\Delta\omega_{0\ GR}\ 2.8218}\ 10^{-3}$ and $J_2 = J_{2\ EPM1988} + \Delta J_2$.
The rounded up result that appears in those articles are not much affected (see our table 3).

In [Pit2001b], the new estimation of $\Delta \overset{\bullet}{\pi} = +0".0055 \pm 0".0085$ is obtained from a fit of EPM2000 numerical ephemeris to radar observational data over the period 1961-1997. But now, $J_{2\ EPM2000} = 2.0\ 10^{-7}$ was assumed and thus, the mean value of the perihelion shift, $\Delta\omega_{EPM2000} \simeq$43".0060, is different from $\Delta\omega_{0\ GR}$.

In [St2000], a value of $J_{2\ DE405} = 2.0\ 10^{-7}$ was assumed. But the estimated perihelion advance given by M. Standish, 42".980 $\pm$ 0".002, contains solely the purely relativistic contribution. The quadrupole moment's, $J_{2\ DE405}$'s, contribution must thus be added.

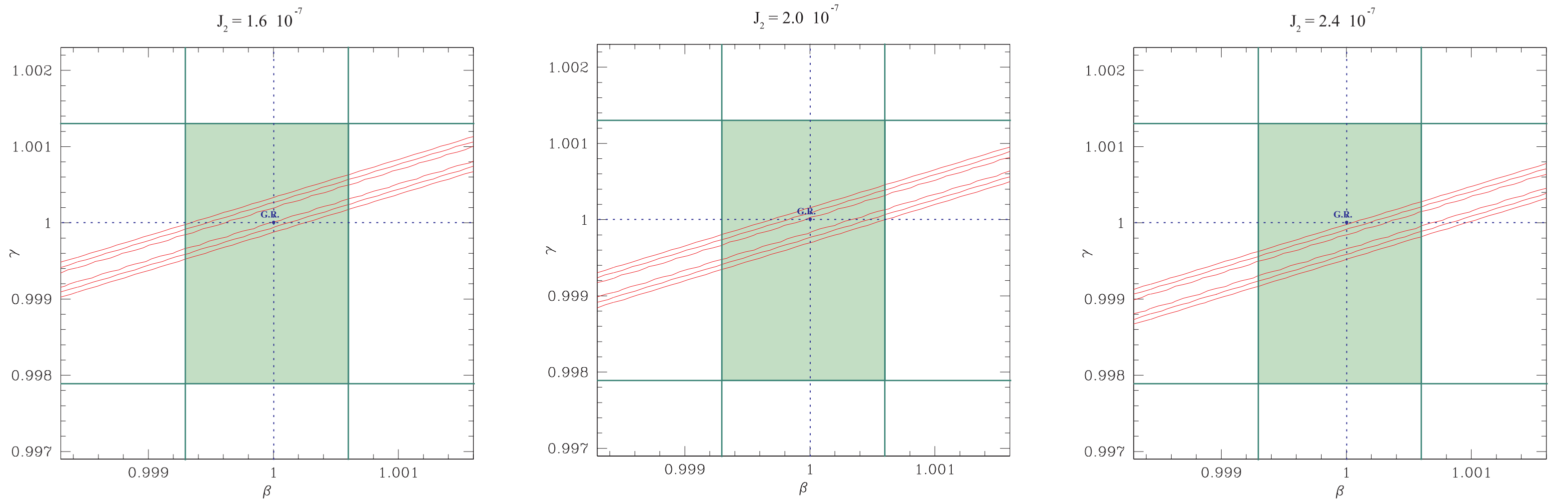

Figure 1. For a given value of J_2, the perihelion advance of Mercury constitutes a test of the P.P.N. parameters β and γ. In the β and γ plane (α set to 1), we have plotted 1σ (the smallest), 2σ and 3σ (the largest) confidence level ellipses. Those are based on the values for Mercury's observed perihelion advance, $\Delta\omega_{obs}$, given in table 1. Notice however that the value given by [Ne1895-1898] as well as those given by [Bre1982a], [Bre1982b] and [Ra1987] have not been taken into account. Indeed, the first cited reference did not contain any error bars estimation; the other ones used an improper method to evaluate $\Delta\omega_{obs}$ (see comments of table 1) and the error bars they provide are truely not realistic ones. Remark also that the position of the ellipses varies according to the value of J_2 chosen; but, their orientation is determined by the combination $(2\alpha^2 + 2\alpha\gamma - \beta)$ that appears in the expression for $\Delta\omega$.
Nevertheless, G.R. is still in the 3σ contours for the allowed theoretical values of J_2 argued by the authors (see section 2.2.2). Fig. 1 a, b, c represent the confidence contours for β and γ, J_2 fixed to its minimum, average and maximum value respectively.

Additional constraints on β and γ (shaded region) can be taken from the Nordtvedt effect and the L.L.R. data (see (12) and (9)).
They allow to determine a portion of the ellipses which constitute the allowed parameter space for β and γ.

- Table 2 -

Estimated values of the Solar quadrupole moment J_2 from solar observations and solar modeling

Year	References	Method	J_2	Critics
1890-1902	[AmSc1905]* [WitDeb1987]*	Direct ground based observation of the solar oblateness at Göttingen (heliometer).	$\leq 4.4\ 10^{-6}$	a, c, d
1891	[WitDeb1987]*	Rotational theory of the Sun by Tisserand	$< 14\ 10^{-6}$	
1909	[WitDeb1987]*	Rotational theory of the Sun by Moulton	$< 20\ 10^{-6}$	
1966	[DiGol1967]* [DiGol1974] [Di et al. 1986]*	Direct ground based observation of the solar oblateness at Princeton (integrated flux from inside till outside the limb).	$(2.37 \pm 0.23)\ 10^{-5}$ $(2.47 \pm 0.23)\ 10^{-5}$	a, d, e f, g, i
	[GolSc1968]	Theory of the solar figure obtained from a rotational law (based upon stability criteria under differential rotation and contemporary surface rotation observations) plus a contemporary density model that are integrated from the center of the Sun till its surface, in order to derive ε at the surface. It is further constrained by a solar evolution model.	$\leq 7.96\ 10^{-5}$	k, l, n, v
1972 1973	[Hi et al.1974]* [HiSt1975]	Direct ground based observation of the solar oblateness using a F.F.T. edge definition, during periods of reduced excess brightness (diameter measurement and excess equatorial brightness monitoring).	$(9.72 \pm 43.4)\ 10^{-7}$	a, d, f, g
(...)				

Year	References	Method	J_2	Critics
(...)				
1979	[Gou1982]	Ground based observation of modes frequency splittings in solar oscillation data allow to infer an internal radial rotation law from which to deduce J_2.	$\geq 1.2\ 10^{-6}$ or $\sim 3.6\ 10^{-6}$	a, k, l, p, t, u v, x
1979	[Hi et al.1982]	Ground based observation of modes frequency splittings in solar oscillation data allow to infer an internal radial rotation law from which to deduce J_2.	$(5.5 \pm 1.3)\ 10^{-6}$	a, k, l, p, t, u v, w
	[UlHa1981]	Theory of the solar figure obtained from a rotational law (based upon a differential rotation model and surface rotation observations) plus a density model that are integrated from the center of the Sun till its surface, in order to derive J_2 at thesurface.	$(1.25 \pm 0.25)\ 10^{-7}$	k, l, n
	[Ki1983]	Theory of the solar figure obtained from a rotational law based upon rigid body like rotation, surface rotation observations and a homogenuous density model. This provides an upper limit for J_2 at the surface.	$< 1.08\ 10^{-5}$	
1979	[Ca et al.1983]	Ground based observation of modes frequency splittings in solar oscillation data allow to infer an internal radial rotation law from which to deduce J_2.	$\geq 1.6\ 10^{-6}$ or $\sim 5.0\ 10^{-6}$	a, p, t, u, v, x
1983	[Di et al.1985] [Di et al. 1986]* [Di et al.1987]*	Direct ground based observation of the solar oblateness during periods of reduced excess brightness, at M^{t} Wilson. (integrated flux from inside till outside the limb).	$(7.92 \pm 0.972)\ 10^{-6}$	a, d, f, g, i, j
(...)				

Year	References	Method	J_2	Critics
(...)				
1984	[Bro et al.1989]	Ground based observation of p-modes frequency splittings in solar oscillation data allow to infer an internal angular rotation law from which to deduce J_2.	$(1.7 \pm 10\%)\ 10^{-7}$	a, o, p, t, u
1984	[Duv et al.1984]	Ground based observation of p-modes frequency splittings in solar oscillation data (...)	$(1.7 \pm 0.4)\ 10^{-7}$	a, u, v
1984	[Di et al. 1986]* [Di et al.1987]*	Direct ground based observation of the solar oblateness during periods of reduced excess brightness. (integrated flux from inside till outside the limb).	$(-1.53 \pm 2.36)\ 10^{-6}$	a, d, f, g, j
1985	[Di et al.1987]*	Direct ground based observation of the solar oblateness during periods of reduced excess brightness. (integrated flux from inside till outside the limb).	$(4.72 \pm 1.53)\ 10^{-6}$	a, d, f, g, j
1986	[Bu1986]	Limits on the solar oblateness from the theory of solar figure given by Roche's and MacLaurin's models. The upper limit of a heavy core is taken to infer J_2.	$\leq 1.1\ 10^{-5}$	
1989	[Del1994]	Analysis of T. Brown's new helioseismic data	$(7.7 \pm 2.1)\ 10^{-6}$	a, p, t, u
1990	[Ma et al.1992]*	Solar Disk Sextant (S.D.S.): a baloon born experiment indirectly measuring the solar angular diameter at a variety of orientations using the F.F.T. edge definition J_2 is then evaluated from the infered solar oblateness and from solar surface angular rotation data.	$(+1.68 \pm 5.70)\ 10^{-6}$	b, d, f, g, y, z, aa, bb, cc, dd, ee
(...)				

Year	References	Method	J_2	Critics
(...)				
1992	[So et al.1994]	Solar Disk Sextant (S.D.S.) (...)	$(0.3 \pm 0.6)\ 10^{-6}$	b, d, f, g, y, z, aa, dd
1992-1994	[El et al.1995]	Ground based observation of p-mode frequency splittings in solar oscillation data obtained from the Birmingham Solar Oscillation Network (BiSON) allow to infermm an internal rotaion law from which to deduce J_2.	$(2.0 \pm 0.5)\ 10^{-7}$	a, l, p, t, ii, jj
1992 1994	[LySo1996]	Solar Disk Sextant (S.D.S.) (...)	$(1.8 \pm 5.1)\ 10^{-7}$	b, d, f, g, y, z, aa, dd ff, gg
1990 1991 1992 1994	[Pa et al.1996]	Solar Disk Sextant (S.D.S.) (1992 and 1994) lead to a measurement of the oblateness (...) which is used with a rotation model in order to evaluate J_2. The surface rotation model is constrained by ground based observations of acoustic p-modes frequency splittings from either the helioseismic network IRIS (1991 -1992) or BiSON (1992-1994).	$(2.22 \pm 0.1)\ 10^{-7}$ IRIS $\updownarrow$ $(2.08 \pm 0.14)\ 10^{-7}$ BISON	S.D.S: b, d, f, g, y, z, aa, dd, ff; BiSON/ IRIS: a, p, hh, ii
(...)				

Year	References	Method	J_2	Critics
(...)				
1993 1994	[Rö et al.1996]	Direct ground based observation of the solar oblateness during periods of reduced excess brightness using the distance between both inflexion points of the limb profile (scanning heliometer provides diameter measurements and excess brightness monitoring).	$(2.57 \pm 2.36)\ 10^{-6}$	a, d, f, g, h
1995 1996	[Pij1998]	Ground based observation of frequency splittings in solar oscillation data obtained from the Global Oscillation Network Group (GONG) (1995-1996) or space observations of oscillations ("a" coefficients) by the Solar Heliospheric Observatory (SoHO) (1996) allow to infer an internal angular rotation law from which to deduce J_2.	$(2.14 \pm 0.09)\ 10^{-7}$ GONG $\updownarrow$ $(2.23 \pm 0.09)\ 10^{-7}$ SoHO $\Rightarrow$ $(2.18 \pm 0.06)\ 10^{-7}$ mean value	a (GONG), k, p
1996	[RozRö1997]	Direct ground based observation of the solar oblateness during periods of reduced excess brightness using the distance between both inflexion points of the limb profile (scanning heliometer provides diameter measurements and excess brightness monitoring).	$(7.57 \pm 15)\ 10^{-7}$	a, d, f, g, h
(...)				

Year	References	Method	J_2	Critics
(...)				
1996 1997	[Ku et al.1998]*	Space observation, by SoHO satellite, of the Sun's full limb position (Michelson Doppler Imager -M.D.I.- experiment) and brightness allow to infer an oblateness from which to deduce J_2 by legendre polynomial fit to the observed limb. (During periods of reduced solar magnetic activity).	$(-12.5 \pm 20.1)\ 10^{-7}$ $\updownarrow$ $(-18.2 \pm 17.6)\ 10^{-7}$ $\Rightarrow$ $(-16.8 \pm 17.3)\ 10^{-7}$ mean value of 1996-1997	d, f, q, r
	[RozBo1998]	Constraints on J_2 from the accurate knowledge of the moon's physical librations, for which the L.L.R. data reach accuracies at the milli- arcsec level.	$\leq 3\ 10^{-6}$	s
	[Rox2000]	Theory of the solar figure obtained from a rotational law (based upon a differential rotation model -deduced from helioseismic inversion- and surface rotation observations) plus a density model that are integrated from the center of the Sun till its surface, to derive J_2 at the surface.	$(2.2125 \pm 0.0075)\ 10^{-7}$	k, l
	[GodRoz1999a] [GodRoz1999b] [GodRoz2000]	Theory of the solar figure obtained from a rotational law (based upon a differential rotation model -deduced from helioseismic data and p-modes frequency splittings obtained by SoHO- and surface rotation observations) plus a density model that are integrated from the center of the Sun till its surface, in order to derive J_2 at the surface.	$(1.6 \pm 0.4)\ 10^{-7}$	k, l, m
	[Ku2001]	Reanalysis of observations.	$2.22\ 10^{-7}$	

Table 2. To each reference corresponds an estimated value of the solar quadrupole moment, the method used to obtain this estimation (solar observations, solar modeling), and some critics we formulate in regards to the method. The year given in the table is the date the observations were made (not the date of the publication).
Notice that some authors[14] only provide the value of solar equatorial excess radius ($\Delta r \equiv r_{equ} - r_{pol}$) in their article. We thus inferred the solar quadrupole moment (J_2) using the following formula [Roz1996] $J_2 = 2/3\ \left(\Delta r - \delta r\right)/r_0$, where $\delta r = 7.8 \pm 2.1$ arc ms [RozRö1997] is the contribution to J_2 due to the surface rotation alone, $\varepsilon \equiv \Delta r/r_0$ is the solar oblateness and $r_0 = 9.6\ 10^5$ arc ms, the solar radius (i.e. the best sphere passing through r_{equ} and r_{pol} [Roz et al.2001]).
The critics or remarks made are the following ones:
(a) Ground based experiments are subject to all kinds of atmospheric perturbations that have to be modeled.
(b) Balloon flights are still subject to some differential refraction due to residual atmosphere (instability) and problems linked to the stability of the pointing instruments.
(c) The maximum value for Δr is taken, as the measured minimum leads to the erroneous prediction of an oblong Sun.
(d) Observations of the oblateness have to be done only during periods of reduced excess brightness in order to be able to deduce the intrinsic visual oblateness from the apparent oblateness obtained with whichever edge definition... until the mechanisms of excess brightness are understood and proper models exists for it.
(e) Did not take into account the solar surface ∇T° which could lead to a difference in brightness indistinguishable from a geometrical oblateness.
(f) The choice of the edge of the sun's definition profoundly influences the sensitivity to excess brightness.
(The F.F.T. -Finite Fourier Transform- edge definition is highly sensitive to the limb darkening shape, but this allow a simultaneous sensitive monitoring of the excess brightness, and detecting local/global active regions without reliance on solar atmosphere models or other observations.)
This leads to discrepancies among the different results obtained for oblateness measurements made during the same period (even with the same instrument!) but using different edge definitions.
(g) The choice of the edge of the sun's definition profoundly influences the appar-

[14]For those authors mentioned with an asterix in the table, the value of J_2 as been inferred from their given value of the oblateness, ε, or excess equatorial radius, Δr.

ent displacement of the Sun's edge attributable to atmospheric seeing.
(The F.F.T. edge definition is less sensitive to this effect than the Dicke Goldenberg integral edge definition.)
(h) Difficulty to correct for the shift of the inflection point.
(i) The stated error on Δr for the 1966 and 1983 experiment is a formal standard deviation. To make allowance for possible seasonal variations in the locally induced atmospheric distortions, the error bars should be increased, possibly to 4 ms. The 1984 and 1985 results are already corrected for this error and thus the derived value of J_2. Notice that 1966 results have often been reinterpreted by the authors leading to different conclusions (see [Di1976]).
(j) Observations in 1983, 1984 and 1985 have been made with a modified instrument (see [Di et al.1985]), by comparison to the 1966 experiment of Dicke-Goldenberg, that automatically excluded data that was contaminated by signals due to substantial facular patches, as well as color dependent brightness signals. The possible existence of a color independent brightness signal is however not taken into account.
(k) Dependent upon the solar density model.
(l) Dependent upon the solar rotation model.
(m) Assumes the same rotation rate for the core and for the radiative zone; but allows $\Omega(r, \theta)$ to vary with the latitude.
(n) Uses a model of internal rotation which is assumed to be uniformly differential (constant on cylinders) through out the convective envelope, and non differential below the convective zone.
(o) No reliable estimates for the uncertainties.
(p) Helioseismic data are limited by the error bars to distances above $0.2\ R_s$, near the surface and near the poles..
(q) A mean limb darkening function is used, while a more realistic model should use a more complex function.
(r) 1996 data set gives noisier results as it was obtained without the active M.D.I. image stabilization system.
(s) Simulations have been performed assuming J_2 constant.
(t) It is difficult to correctly identify individual modes of oscillation for all spatial scales. Moreover, the oscillations must be adequately long lived.
(u) Oscillations, as observed from the ground, can not provide a good measure of the rotation rate near the solar pole, because foreshortening limits the viewing region.
(v) Assume that the rotational frequency is independent of the latitude.
(w) Maximum consistent with the stability of the Sun.

(x) Some fits to the data produced rotation curves, $\Omega(r)$, that were highly unphysical. The given value for J_2 corresponds to the smoothest curve that fits the data.
(y) A small quantity, the separation between 2 solar limb images, is measured, instead of the full solar diameter. This enhances the precision with respect to the techniques that measure the full diameter directly.
(z) The instrument scale can be calibrated for any measurement.
(aa) The quantity measured is located near the optical axis of the instrument (unlike for direct diameter measurements), where the optical system is optimal.
(bb) No true perpendicular diameters are measured (polar and equatorial). The resulting J_2 is thus probably less than its real value.
(cc) The resulting oblateness is $\sim 30^{\circ}$ offset from the polar-equator position.
(dd) Solar photospheric T° and ∇T° may be a function of the activity solar cycle, and so, the use of F.F.T. definitions into data reduction from the S.D.S experiment would introduce systematic errors.
(ee) Gravitational distortions (a non constant wedge angle) of the instrument exist that were avoided in the next balloon flights (1992-1994).
(ff) S.D.S. experiments were made on 2 days, 2 years apart (1992-1994) rather than continuously over a period of many years. Moreover, there were no observations of solar surface rotation available between 1992-1994. Thus, the large number of observations did not allow to lower the uncertainties.
(gg) Uses a simple model of internal rotation of the Sun as constant angular rotation on cylinders or on cones.
(hh) BiSON's helioseismic data imply a solar rotation law which is not compatible with that inferred from IRIS's.
(ii) The rotation model does not take into account helioseismologic observations made at different latitudes.
(jj) It was not possible to find a rotation model that reproduced both the splitting data reported by [El et al.1995] and the data from the Big Bear Solar Observatory (B.B.S.O.)

Table 3. This table follows table 1. To each reference corresponds an inferred value of the solar quadrupole moment, in the setting of G.R., using the perihelion shift of Mercury.
Notice that concerning references [Kr et al.1986], [Kr et al.1993], [Pit1993] and [Pit2001b], the value of the solar quadrupole moment is calculated according to the formula given in the comments of table 1.

- Table 3 -
Inferred solar quadrupole moment
from the perihelion shift of Mercury, assuming G.R.

References	Inferred Quadrupole Moment $\frac{J_2}{(10^{-7})} = \frac{\Delta\omega_{obs}-\Delta\omega_{0\ GR}}{\Delta\omega_{0\ GR}\ 2.8218\ 10^{-4}}$
[Ne1895-1898]	$\sim +32.1$
[Cl1943]	-11.6 ± 83.3
[Cl1947]	-33.9 ± 79.2
[Dun1958]	$+9.8 \pm 36.3$
[Way1966]	$+79.9 \pm 33.8$
[Sh et al.1972]	$+13.9 \pm 24.7$
[MoWar1975]	-89.1 ± 41.2
[Sh et al.1976]	$+10.6 \pm 17.3$
[An et al.1978]	$+26.3 \pm 16.5$
[Bre1982a] [NaRa1985]	$+199.4 \pm 4.1$
[Bre1982b] [Ra1987]	$+187.1 \pm 4.1$
[Kr et al.1986]:	
EPM1988	-12.3 ± 10.0
DE200	-17.1 ± 9.7
[Ra1987]	$+205.2 \pm 7.4$
[An et al.1987]	-5.0 ± 16.5
[An1991]	-3.4 ± 16.5
[An1992]	$+12.3 \pm 11.5$
[Kr et al.1993]:	
EPM1988	$+0.33 \pm 5.03$
DE200	-0.25 ± 5.03
[Pit1993]:	
EPM1988	-1.40 ± 4.29
DE200	-0.91 ± 4.29
[St2000]:	
DE405	$+1.90 \pm 0.16$
[Pit2001b]:	
EPM2000	$+2.453 \pm 0.701$

- Table 4 - Inferred solar quadrupole moment from the perihelion shift of Icarus, assuming G.R.	
References	Inferred Quadrupole Moment J_2 (by fitting the parameters)
[LiNu1969]	$(+1.8 \pm 2.0)\ 10^{-5}$
[La1992]	$(-0.65 \pm 5.84)\ 10^{-6}$ or $\leq 2\ 10^{-5}$

Table 4. To each reference corresponds an inferred value of the solar quadrupole moment, in the setting of G.R., using the perihelion shift of Icarus.

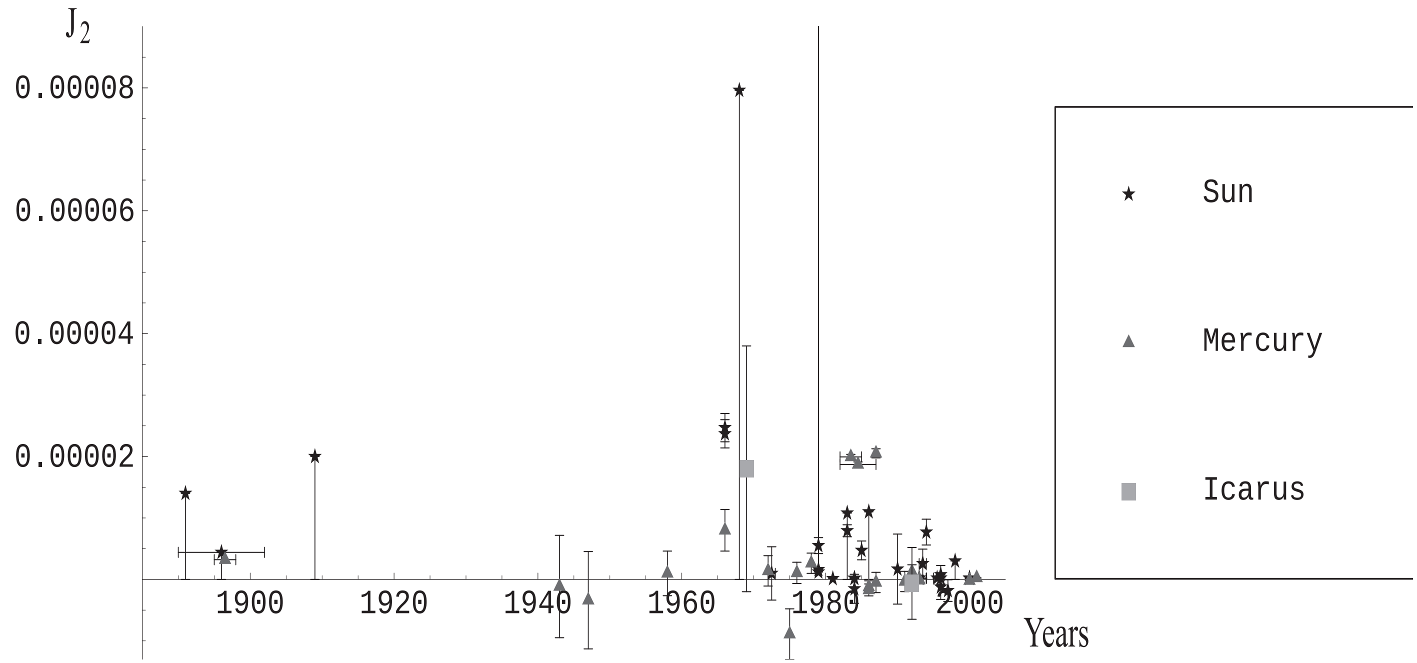

Figure 2. Different estimated values of the solar quadrupole moment, J_2, versus the date when the respective observations were made. There are 3 types data points: values estimated from solar models and observations (table 2), values infered from the perihelion shift of Mercury (table 3) and those obtained from Icarus' (table 4).

- Table 5 -

Proposed space missions dedicated to γ

References	Method	Mission	Expected precision on γ
[BeGi1998] [Ie et al.1999]	doppler measurement of the Solar gravitational deflection, the first time this method is used	Cassini launched in 1997, experiment in 2002-2003	$10^{-4} - 10^{-5}$
[Fi et al.1995], [GAIA2000] http://einstein.standford.edu	relativity gyroscope experiment, geodetic precession measurement	Gravity Probe B (2002)	$6\ 10^{-5}$
[Bepi2000], [Tu et al.1996] http://www.estec.esa.nl/spdwww/future/html/meo2.htm	output of orbit determination, time delay and doppler shift measurements (see section 4.3)	Mercury Orbiter, within BepiColombo (2007/2009)	$2.5\ 10^{-6}$
[Re1999] http://www.cnes.fr/WEB_UK/activities/index.htm, see in “Understanding the universe”,“Fund. Phys.”	Projet d’Hologe Atomique par refroidissement d’Atomes en Orbite (PHARAO clock), a swiss hydrogen maser clock to provide a long term frequency standard, associated with the IIS, to form the Atomic Clock Ensemble in Space (ACES)	International Space Station (IIS) (2004/2005)	$1\ 10^{-5}$
[Re1999]	Time delay / light deflection measurements (see section 4.3)	Solar Orbit Relativity Test (SORT) (after 2010)	$1\ 10^{-7}$

Table 5. Direct measurement of γ: Some space experiments dedicated to γ.

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